Int. J. Heat Mass Transfer. Vol. 16, pp. 213–216. Pergamon Press 1973. Printed in Great Britain

SHORTER COMMUNICATIONS

HEAT TRANSFER WITH A MOVING BOUNDARY: APPLICATION TO FLUIDIZED-BED COATING OF THIN PLATES

N. ABUAF and C. GUTFINGER

Laboratory for Coating Technology, Department of Mechanical Engineering, Technion–Israel Institute of Technology, Haifa, Israel

(*Received* 2 *December* 1971 *and in revised form* 17 *May* 1972)

NOMENCLATURE

A_w, surface area of object [ft^2];
Bi, Biot number, $h\delta_F/k_c$;
c_c, specific heat of coating material [Btu/lb°F];
c_w, specific heat of object [Btu/lb°F];
h, convective heat-transfer coefficient [Btu/hft^2°F];
k_c, thermal conductivity of coating material [Btu/hft°F];
L, half thickness of object [ft];
m_w, mass of object [lb], $LA_w\rho_w$;
t, time [h];
T, temperature [°F];
T_m, softening temperature of coating material [°F];
T_w, object temperature [°F];
T_{w0}, initial object paramater [°F];
x, position coordinate measured from wall [ft];
Y, dimensionless parameter, δ_F/L;
Z, dimensionless parameter, $\rho_c c_c/\rho_w c_w$.

Greek symbols

δ, coating thickness [ft];
δ_F, final coating thickness for constant wall temperature [ft], $k_c(T_{w0} - T_m)/h(T_m - T_\infty)$;
δ_f, final coating thickness for falling object temperature [ft];
Δ, dimensionless coating thickness, δ/δ_F;
Δ_f, dimensionless final coating thickness, δ_f/δ_F;
θ, dimensionless temperature, $(T - T_\infty)/(T_{w0} - T_\infty)$;
τ, dimensionless time, $k_c \cdot t/\rho_c c_c \delta_F^2$;
ρ_c, density of coating material [lb/ft^3];
ρ_w, density of object [lb/ft^3].

INTRODUCTION

WHEN a hot object is dipped in a bed of fluidized plastic powder, a film of fused plastic coating will be formed on its surface. The coating thickness depends on the object temperature, the fusion temperature of the powder, the immersion time in the bed, the physical properties of the object and the powder, as well as on the heat content of the object.

If the object possesses a very large heat content, it can be considered as an infinite heat source, and its temperature could be taken as constant during the coating process. The case of constant wall temperature fluidized bed coating was analyzed by Gutfinger and Chen [1, 2]. Methods for determining the final coating thickness, as a function of the different parameters and time, were devised.

In reality, the object possesses a finite heat capacity and its temperature drops during the coating process. This implies that the previous results obtained with a constant wall temperature will be the upper bound on the coating thickness, and that in reality smaller coating thicknesses will be obtained at finite times.

In this investigation the problem of fluidized bed coating of low heat content objects is analysed as a heat-transfer problem with a moving boundary. Thus it is an extension of the previously mentioned analysis of the constant wall temperature process. The final results obtained enable one to calculate the coating thickness of low heat content objects.

ANALYSIS OF THE PROBLEM

Consider a flat plate with an area A_w and half thickness L, which is dipped vertically in a fluidized bed. The object is initially at a temperature, T_{w0}, which is higher than the softening temperature of the coating material, T_m. The plastic coating material in contact with the object surface will melt and begin to form a layer on the plate. The process now involves the transfer of heat from the plate to the continuously growing film, and then into the fluidized bed.

Usually the object is metallic possessing a high thermal conductivity and a finite heat capacity, therefore its temperature will remain uniform but decrease with time, i.e. the body is treated as a lumped parameter system. Heat from the plate is transferred through the coating film whose properties, ρ_c, c_c, k_c, are assumed to remain constant during the process. The surface temperature of the coating film is assumed to be constant and equal to the melting or softening temperature of the coating polymer, T_m. It is also assumed that the molten plastic does not flow down the wall. The latter assumption is reasonable in view of the high viscosity of molten polymers and short duration of the coating process. Although the rate of heat convected from the coating surface into the bed is obviously dependent on the fluidization conditions and the temperature gradient, in this analysis the heat transfer coefficient between the plate and the fluidized bed is assumed to be independent of location and direction, and is taken as constant during the coating. The temperature within the fluidized bed, T_∞, is assumed to be uniform and constant. With these assumptions, the one dimensional heat conduction problem with a moving boundary and variable wall temperature is described by the following equations:

$$\rho_c c_c \frac{\partial T}{\partial t} = k_c \frac{\partial^2 T}{\partial x^2} \tag{1}$$

$$T(0,0) = T_{w0} \tag{2}$$

$$T(0,t) = T_w(t) \tag{3}$$

$$T(\delta,t) = T_m \tag{4}$$

$$\frac{m_w c_w}{A_w}(T_{w0} - T_w(t)) = ht(T_m - T_\infty) + \rho_c c_c \int_0^{\delta(t)} (T - T_\infty)\,\mathrm{d}x \tag{5}$$

$$-k_c \frac{\partial T}{\partial x}\bigg|_{x=\delta} = h(T_m - T_\infty) + \rho_c c_c (T_m - T_\infty)\frac{\mathrm{d}\delta}{\mathrm{d}t} \tag{6}$$

$$\delta(0) = 0. \tag{7}$$

Equation (6) is the heat balance at the surface of the coating film. The latent heat of fusion of the polymer is not taken into account. This is due to the fact that polymers with a crystalline structure, that possess a heat of fusion, are unsuitable for coating. On the other hand, for polymers not possessing a latent heat of fusion, the heat capacity in the softening region is not constant and one, therefore, has to work with heat capacities averaged over the temperature range studied. For cases where the latent heat cannot be neglected, there is no inherent difficulty to include it in the present analysis. However, this would add an additional parameter to the problem· Equation (5) expresses the fact that the heat loss by the body during the time interval $(0, t)$ is equal to the heat transferred to the fluidized bed, plus the heat consumed in bringing the coating film temperature from its initial value, T_∞, to its final value $T(x, t)$.

We rewrite now the equations in dimensionless form by defining a dimensionless coordinate, time, temperature, and coating thickness, respectively: $\xi = x/\delta$; $\tau = k_c t/\rho_c c_c \delta_F^2$; $\theta = (T - T_\infty)/(T_{w0} - T_\infty)$; $\Delta = \delta/\delta_F$.

The final coating thickness, δ_F, is the one obtained from the solution of the coating problem for a constant object, or wall temperature. The expression for this final coating thickness was derived (1), (2), as:

$$\delta_F = \frac{k_c}{h}\frac{T_{w0} - T_m}{T_m - T_\infty}. \tag{8}$$

In the present investigation one does not assume a constant object temperature. Here it will drop with time as the film thickness grows. Thus, the final thickness in our case will be lower than the one given by equation (8). The latter can be considered to be the upper bound. How closely this upper bound is approached with a falling wall temperature will be expressed by the dimensionless thickness, Δ.

Equations (1)–(7) are rewritten in dimensionless form as:

$$\Delta^2 \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\xi}{2}\cdot\frac{\mathrm{d}\Delta^2}{\mathrm{d}\tau}\cdot\frac{\partial \theta}{\partial \xi} \tag{9}$$

$$\theta(0,0) = 1 \tag{10}$$

$$\theta(0,\tau) = \theta_w \tag{11}$$

$$\theta(1,\tau) = \theta_m \tag{12}$$

$$\theta_w = 1 - ZYBi\theta_m\tau - ZY\Delta\int_0^1 \theta\,\mathrm{d}\xi \tag{13}$$

$$-\frac{\partial \theta}{\partial \xi}(1,\tau) = Bi\Delta\theta_m + \frac{1}{2}\theta_m\frac{\mathrm{d}\Delta^2}{\mathrm{d}\tau} \tag{14}$$

$$\Delta(0) = 0 \tag{15}$$

where $Bi = h\delta_F/k_c$ is the Biot number based on δ_F, and $Z = \rho_c c_c/\rho_w c_w$, $Y = \delta_F/L$ are dimensionless parameters. Substituting δ_F from equation (8), one can express the Biot number as:

$$Bi = (\theta_{w0} - \theta_m)/\theta_m. \tag{16}$$

Equation (9), first derived by Landau [3], points out quite clearly the non-linear characteristic of the differential equation for the heat transfer problem with a moving boundary.

The above set of differential equations (9)–(15) is solved by applying an integral technique similar to the one used by Goodman [4], Savino and Siegel [5, 6], Gutfinger and Chen [1, 2] and Elmas [7].

Integrating twice equation (9) with the proper boundary conditions and following the method of Siegel and Savino [5] one obtains

$$\theta(\xi,\tau) = \theta_w(0,\tau) - Bi\theta_m\Delta\xi - \frac{1}{2}\frac{\mathrm{d}\Delta^2}{\mathrm{d}\tau} \times \left[2\int_0^\xi \xi\theta\,\mathrm{d}\xi + \xi\int_\xi^1 \theta\,\mathrm{d}\xi\right] - \Delta^2\frac{\partial}{\partial\tau}\left[\int_0^\xi \xi\theta\,\mathrm{d}\xi + \xi\int_\xi^1 \theta\,\mathrm{d}\xi\right]. \tag{17}$$

This equation is the expression for the instantaneous temperature distribution in the coating film. Rewriting equation (17) for the film surface ($\xi = 1$) and integrating with respect to time we obtain:

$$\Delta^2 = \frac{\int_0^\tau (\theta_w - \theta_m - Bi\theta_m \Delta)\,d\tau}{\int_0^1 \xi\theta\,d\xi}. \tag{18}$$

Equation (18) yields an expression for the change of the coating thickness with time as a function of the wall temperature, θ_w, and the film temperature profile, θ. These are given by equations (13) and (17), respectively. The problem is now reduced to the solution of three simultaneous equations, (13), (17) and (18), which will provide expressions for $\theta = \theta(\xi,\tau)$, $\theta_w = \theta_w(\tau)$, and $\Delta = \Delta(\tau)$. A similar approach to fluidized bed coating has been taken by Elmas [7]. However, Elmas failed to realize the coupling between the heat transfer inside the coating film to that of the object. In his case instead of equation (13), he assumes an exponential decay of the wall temperature with the square root of time, and a linear temperature variation in the coating. Even with these simplifications Elmas stops short of solving his equations except for the special case of constant wall temperature previously reported by Gutfinger and Chen [1].

The system of three simultaneous equations seems quite difficult to solve analytically. In equations (18) and (13) giving the coating thickness and the wall temperature, respectively, the film temperature distribution is under the integral sign. Thus, one will not err appreciably in representing it by a second degree polynominal

$$\theta = a + b(1-\xi) + c(1-\xi)^2. \tag{19}$$

The coefficients a, b and c, which are functions of time, are evaluated with the help of the boundary conditions resulting in the following:

$$\begin{aligned}\theta = \theta_m + \frac{1}{2}\theta_m \Bigg\{ & Bi\Delta - 2 + \sqrt{\left[(Bi\Delta - 2)^2 + 8\frac{\theta_w - \theta_m}{\theta_m}\right]}\Bigg\}(1-\xi) \\ - \frac{1}{2}\theta_m \Bigg\{ & Bi\Delta - 2 + \sqrt{\left[(Bi\Delta - 2)^2 + 8\frac{\theta_w - \theta_m}{\theta_m}\right]} - 2\frac{\theta_w - \theta_m}{\theta_m}\Bigg\} \\ & \times (1-\xi)^2. \end{aligned} \tag{20}$$

This temperature profile is substituted in equations (13) and (18), integrations performed, and the following final equations are obtained for the wall temperature and coating thickness

$$\begin{aligned}\theta_w = 1 - ZYBi\theta_m\tau - \frac{ZY\Delta}{12} \times \Bigg\{ & 6\theta_m + \theta_m Bi\Delta + 4\theta_w + \theta_m \sqrt{\left[(Bi\Delta - 2)^2 + 8\frac{\theta_w - \theta_m}{\theta_m}\right]}\Bigg\}\end{aligned} \tag{21}$$

and

$$\begin{aligned}\Delta^2 \Bigg\{ & 8\theta_m + \theta_m Bi\Delta + 2\theta_w + \theta_m \sqrt{\left[(Bi\Delta - 2)^2 + 8\frac{\theta_w - \theta_m}{\theta_m}\right]}\Bigg\} \\ & = 24\int_0^\tau (\theta_w - \theta_m - Bi\theta_m\Delta)\,d\tau. \end{aligned} \tag{22}$$

The set of two simultaneous equations for θ_w and Δ was solved by an iterative numercial method using an IBM 370/165 computer.

RESULTS

Typical results are plotted in Figs. 1 and 2. The remainder of the figures were eliminated in the process of converting this paper into a shorter communication upon the request of a space-conscious reviewer.

Figure 1 provides the coating histories at different coating conditions, whereas Fig. 2 shows the effect of the coating parameters on the final thickness and wall temperature.

Inspecting equations (21) and (22) that define the problem, one notes that the dimensionless coating thickness, Δ, is a function of dimensionless time, τ, melting temperature, θ_m, and the product, ZY

$$\Delta = \Delta(\tau; \theta_m, ZY). \tag{23}$$

When plotting Δ vs. τ, only one of the parameters can be varied while the other one has to be kept constant. The product ZY which appears in equation (21) always comes together. Expressed in terms of the parameters of the problem, it can be written as:

$$ZY = \frac{\rho_c c_c}{\rho_w c_w} \cdot \frac{k_c}{hL} \cdot \frac{\theta_{w_0} - \theta_m}{\theta_m}. \tag{24}$$

The reason for the parameters Z and Y appearing together is due to the fact that the coated object was taken as a lumped parameter system.

Figure 1 plots the dimensionless coating thickness and wall temperature as a function of the dimensionless time, τ, for $\theta_m = 0{\cdot}5$ and the product ZY as a parameter.

As seen, for values of ZY smaller than 0·1 the final thickness can be approximated by the constant temperature solution with an error of less than 20 per cent.

Figure 2 provides a plot of the dimensionless final thickness and temperature as a function of ZY, with θ_m as a parameter. This figure is the one that the practicing coating technologist will be mostly interested in, as it shows the highest coating thickness possible for a given set of coating parameters as well as the drop in wall temperature at the point where the final thickness is achieved.

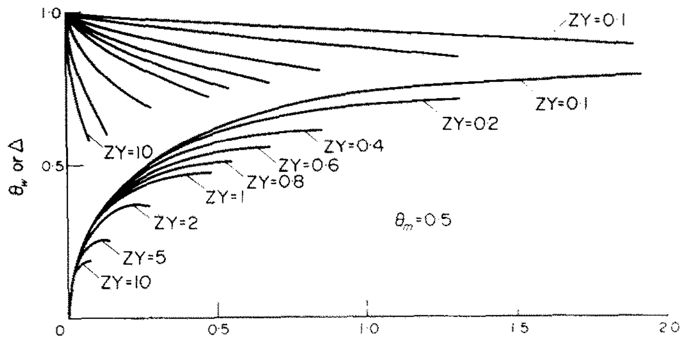

FIG. 1. Dimensionless coating thickness, Δ, and dimensionless wall temperature, θ_w, as a function of dimensionless time, τ, for various values of the parameter ZY, $\theta_m = 0{\cdot}5$. The upper curves represent θ_w, and the lower ones Δ.

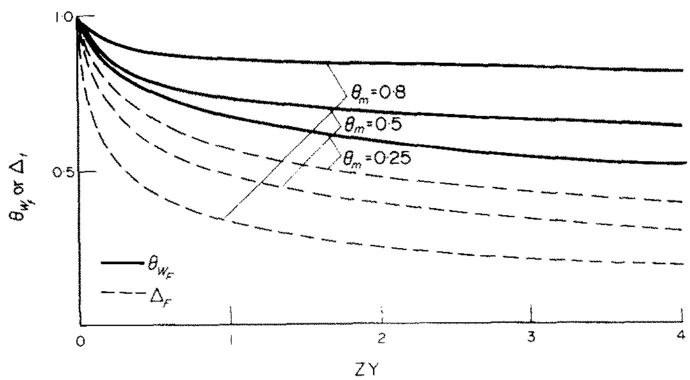

FIG. 2. Plot of final dimensionless coating thickness, Δ_f, and of final dimensionless wall temperature, θ_{w_f}, vs. dimensionless parameter, ZY, for various values of the dimensionless melting temperature θ_m.

REFERENCES

1. C. GUTFINGER and W. H. CHEN, Heat transfer with a moving boundary-application to fluidized bed coating, *Int. J. Heat Mass Transfer* **12**, 1097–1108 (1969).
2. C. GUTFINGER and W. H. CHEN, An approximate theory of fluidized bed coating, *Chem. Engng Prog. Symp. Ser.* 101, **66**, 91–100 (1970).
3. H. G. LANDAU, Heat conduction in a melting solid, *Q. Appl. Math.* **8**, 81 (1950).
4. T. R. GOODMAN, The heat-balance integral and its application to problems involving a change of phase, *Trans. Am. Soc. Mech. Engrs* **80**, 335–342 (1958).
5. R. SIEGEL and J. M. SAVINO, An analysis of the transient solidification of a flowing warm liquid on a convectively cooled wall, Proc. 3rd Int. Heat Transfer Conf. Vol. 4, 141–151, ASME (1966).
6. J. M. SAVINO and R. SIEGEL, An analytical solution for solidification of a moving warm liquid onto an isothermal cold wall, *Int. J. Heat Mass Transfer* **12**, 803–809 (1969).
7. M. ELMAS, On heat transfer with moving boundary, *Int. J. Heat Mass Transfer* **13**, 1625–1627 (1970).